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# An Analysis of Thermal Response Factors & How to Reduce Their Computational Time Requirement

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## SUMMARY

This paper presents an in-depth study of the Thermal Response Factor Method enhanced by G. P. Mitalas and J. G. Arseneault. A concise analysis of the theoretical mathematics and physical interpretation of response factors will be presented. It will be shown how a new root calculating procedure was developed to substantially reduce the computational time requirement of response factors by an average of 88.5%. Comparison testing between the original and the new procedures will be presented to validate the new root procedure.

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## INTRODUCTION

The Thermal Response Factor Method is recognized by the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) as the current state-of-the-art technique to use for the computer simulation of hourly transient heat conduction analysis. This method calculates hourly heat transfer coefficients (response factors) by using the material properties of a particular planar construction. These response factors are then combined with an appropriate temperature history curve to yield a series solution for the heat flux at a given time. This Response Factor Method is the heart of many current energy analysis programs (such as NASA's ENERGY COST ANALYSIS PROGRAM - NECAP).

This paper is an in-depth study of the Response Factor Method enhanced by G. P. Mitalas and J. G. Arseneault, and of the RESPONSE FACTOR PROGRAM (RESFAC<sup>1</sup>). As this research progressed, it became quite apparent that the calculation of thermal response factors was being handled in a 'black box' manner. To rectify this, a concise analysis of the theoretical mathematics and physical interpretation will be presented.

The second part of this paper will present an investigation of the response factor's computational time requirement which has been criticized for taking too long. In particular, the root finding procedure has been singled out as the main culprit. To determine the validity of this criticism, the RESFAC code was analyzed and test cases con-

sisting of various planar constructions were run. No basis for the computational time criticism could be found in the test cases' results. (See TABLE I)

However, the analysis of the FORTRAN code revealed several portions of inefficient coding. Moreover, it was determined that a faster, more efficient, and more accurate root calculating procedure could be developed. Upon cleaning up the code and installing a new root procedure, the modified RESFAC was benchmarked using the test cases mentioned previously. The validation testing shows a substantial reduction in RESFAC's computational time requirement. (see TABLE I)

# NOMENCLATURE

$c$  : specific heat (BTU/LB-°F)  
 $D_i$  : density of material layer  $i$  (LB/FT<sup>3</sup>)  
 $f$  : heat flux function (BTU/HR-FT<sup>2</sup>)  
 $\bar{F}$  : Laplace transform of heat flux function  
 $k$  : conductivity (BTU/HR-FT-°f)  
NRT : number of response factor terms calculated  
 $P$  : Laplace transform parameter  
 $QI$  : inside surface heat flux (BTU/HR-FT<sup>2</sup>)  
 $QO$  : outside surface heat flux (BTU/HR-FT<sup>2</sup>)  
 $R_i$  : thermal resistance of material layer  $i$  (HR-FT<sup>2</sup>-°F/BTU)  
RN : number of real, positive roots calculated  
 $R_t$  : total thermal resistance of all material layers  
(HR-FT<sup>2</sup>-°F/BTU)  
 $SH_i$  : specific heat of material layer  $i$  (BTU/LB-°F)  
 $t$  : time  
 $TI$  : inside surface temperature (°F)  
 $TO$  : outside surface temperature (°F)  
 $u$  : Heaviside unit function  
 $U$  : overall heat transfer coefficient (BTU/HR-FT<sup>2</sup>-°F)  
 $v$  : temperature function (°F)  
 $\bar{V}$  : Laplace transform of temperature function  
 $W$  : real, positive roots of  $B(W_k)=0.0$ ,  $k=1, 2, \dots$  NRT  
 $X, Y, Z$  : thermal response factor series (BTU/HR-FT<sup>2</sup>-°F)  
 $XK_i$  : conductivity of material layer  $i$  (BTU/HR-FT-°F)  
 $XL_i$  : thickness of material layer  $i$  (FT)  
 $\alpha$  : diffusivity (FT<sup>2</sup>/HR)

$\eta$  : number of material layers

$\phi$  : pulse function

$\bar{\phi}$  : Laplace transform of pulse function

$\delta$  : time increment

$\rho$  : density (LB/FT<sup>3</sup>)

BETA<sub>i</sub> : reciprocal of the square root of Fourier number  
for material layer i

$$\text{BETA}_i = \text{XL}_i * \left( \frac{D_i * \text{SH}_i}{\text{XK}_i * \delta} \right)^{1/2}$$

A, B, C, D: overall conduction matrix elements where

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} * \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} * \dots * \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix};$$

$\eta$  = number of material layers;  $a_i$ ,  $b_i$ ,  $c_i$ , and  $d_i$   
are the conduction matrix elements for each  
material layer  $i = 1, 2, \dots, \eta$

dA, dB, dC, dD: overall derivatives of the conduction  
matrix elements where

$$d \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} da_1 & db_1 \\ dc_1 & dd_1 \end{pmatrix} * \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} * \dots * \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix} +$$

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} * \begin{pmatrix} da_2 & db_2 \\ dc_2 & dd_2 \end{pmatrix} * \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix} * \dots * \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix} + \dots +$$

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} * \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} * \dots * \begin{pmatrix} a_{n-1} & b_{n-1} \\ c_{n-1} & d_{n-1} \end{pmatrix} * \begin{pmatrix} da_n & db_n \\ dc_n & dd_n \end{pmatrix}$$



NOTES

## ANALYZING RESPONSE FACTORS

### Physical Interpretation

Thermal response factors can be defined as time dependent heat transfer coefficients which depict an object's action upon a particular outside surface/inside surface temperature difference over a period of time. This action is composed of three terms--the X, Y, and Z response factors.

Using the heat flux equation  $Q = U * \Delta T$  along with the response factors, the time dependent heat flux values for the planar construction shown in Figure 1 can be obtained.

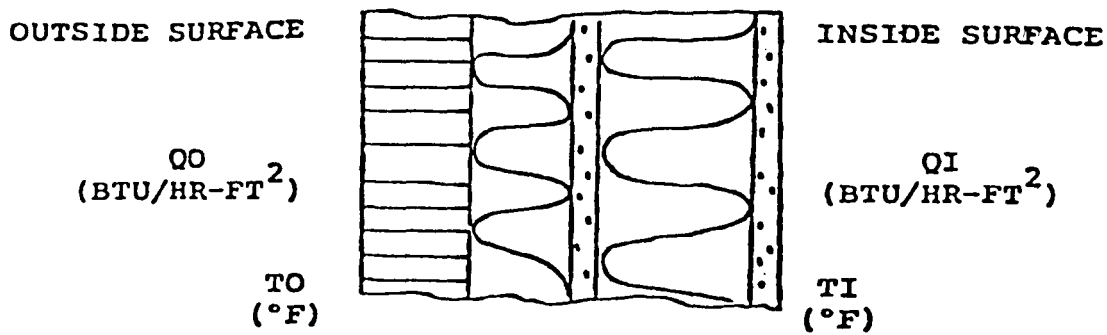


FIGURE 1

The outside surface heat flux ( $QO$ ) at any time  $t$  is calculated by<sup>9</sup>:

$$QO_t = \sum_{i=0}^{NRT} (TO_{t-i} * X_i) - \sum_{i=0}^{NRT} (TI_{t-i} * Y_i)$$

where NRT is the number of response factor terms calculated,  $TO_{t-i}$  and  $TI_{t-i}$  are outside surface and inside surface temperatures at times  $t-i$  hour.

The inside surface heat flux (QI) at any time t is calculated by<sup>9</sup>:

$$QI_t = \sum_{i=0}^{NRT} (TO_{t-i} * Y_i) - \sum_{i=0}^{NRT} (TI_{t-i} * Z_i)$$

In order for the steady state heat conduction situation<sup>9</sup>

$$QO_t = QI_t = U(TO - TI)$$

$$TO = TO_t \quad \text{for any time } t$$

$$TI = TI_t \quad \text{for any time } t$$

to be satisfied, the individual summation of the X, Y, and Z response factors must equal the overall heat transfer coefficient U<sup>9</sup>:

$$U = \left| \sum_{i=1}^{\infty} X_i \right| = \left| \sum_{i=1}^{\infty} Y_i \right| = \left| \sum_{i=1}^{\infty} Z_i \right|$$

### Mathematical Derivation

For a homogeneous, isotropic solid whose thermal conductivity is independent of temperature, the one dimensional equation of conduction of heat with no heat generation is:

$$(1) \quad \frac{\partial^2 v}{\partial x^2} - \frac{1}{\alpha} \frac{\partial v}{\partial t} = 0$$

where  $\alpha$ , the diffusivity,  $= \frac{k}{\rho c}$  and  $k$  is the conductivity,  $\rho$  is density, and  $c$  is specific heat.

For a planar construction that experiences a steady, periodic temperature  $v(t)$ , the boundary conditions are:

$$v(t) = v_0 \phi(t) \text{ for the surface at } x = 0$$

$$v(t) = v_L \phi(t) \text{ for the surface at } x = L$$

where  $L$  is the thickness of the construction.

$\phi(t)$  is a pulse function of period  $2\delta$  ( $\delta$  = discrete hourly time interval) and has the values:

$$(2) \quad \begin{cases} \phi = \frac{1}{\delta} & , 0 < t \leq \delta \\ \phi = \frac{1}{\delta} - \frac{2}{\delta} [u(t - \delta)] & , \delta < t \leq 2\delta \\ \phi = \frac{1}{\delta} - \frac{2}{\delta} [u(t - \delta)] + \frac{1}{\delta} [u(t - 2\delta)] & , t > 2\delta \end{cases}$$

where  $u$  is the Heaviside unit function.<sup>3</sup>

The heat flux can then be expressed as:

$$(3) \quad f_x = -\frac{1}{R} \phi(t) \frac{\partial v}{\partial x}$$

where  $R$ , the resistivity,  $= \frac{1}{k}$ .

The Laplace transforms of equations (1), (2), and (3) are now needed. Using the Laplace transform equation<sup>2</sup>

$$\bar{S} = \int_0^{\infty} e^{-Pt} S(t) dt,$$

where  $P$  is the Laplace transform parameter, the solutions are:

$$(4) \quad \frac{d^2 \bar{V}}{dx^2} - \frac{P}{\alpha} \bar{V} = 0$$

$$(5) \quad \begin{cases} \bar{\phi} = \frac{1}{\delta P} & , 0 < t \leq \delta \\ \bar{\phi} = \frac{1}{\delta P} - \frac{2e^{-P\delta}}{\delta P} & , \delta < t \leq 2\delta \\ \bar{\phi} = \frac{1}{\delta P} - \frac{2e^{-P\delta}}{\delta P} + \frac{e^{-2P\delta}}{\delta P} & , t > 2\delta \end{cases}$$

$$(6) \quad \bar{F} = -\frac{1}{R} \bar{\phi} \frac{d\bar{V}}{dx}$$

The solutions (only the real parts) to equations (4) and (6) are:

$$(7) \quad \begin{cases} \bar{V}_x \bar{\phi} = \bar{V}_0 \bar{\phi} \cos qx + \frac{\bar{F}_0 R \sin qx}{q} \\ \bar{F}_x = -\frac{\bar{V}_0 \bar{\phi} q \sin qx}{R} + \bar{F}_0 \cos qx \end{cases}$$

where  $q = \sqrt{\frac{P}{\alpha}}$ . This solution can then be expressed in matrix form:

$$(8) \quad \begin{pmatrix} \bar{V}_x \bar{\phi} \\ \bar{F}_x \end{pmatrix} = \begin{pmatrix} \cos qx & \frac{R}{q} \sin qx \\ -\frac{q}{R} \sin qx & \cos qx \end{pmatrix} \begin{pmatrix} \bar{V}_0 \bar{\phi} \\ \bar{F}_0 \end{pmatrix}$$

In terms of heat flux, the matrix can be manipulated into:

$$(9) \quad \begin{pmatrix} \bar{F}_0 \\ \bar{F}_x \end{pmatrix} = \begin{pmatrix} \frac{-q \cos qx}{R \sin qx} & \frac{q}{R \sin qx} \\ \frac{-q}{R \sin qx} & \frac{q \cos qx}{R \sin qx} \end{pmatrix} \begin{pmatrix} \bar{\phi} \bar{V}_0 \\ \bar{\phi} \bar{V}_x \end{pmatrix}$$

When the planar construction is composed of varying material layers, the matrix (8) would be represented as:

$$(10) \begin{pmatrix} \bar{V}_L \bar{\phi} \\ \bar{F}_L \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \dots \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix} \begin{pmatrix} \bar{V}_0 \bar{\phi} \\ \bar{F}_0 \end{pmatrix}$$

$$= \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \bar{V}_0 \bar{\phi} \\ \bar{F}_0 \end{pmatrix}$$

where  $\begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix}$  for  $i = 1, 2, \dots, n$  (number of layers)

is each layers 2\*2 matrix and  $L$  is the thickness of the construction. Note that this solution assumes that there is no contact resistance between the layers. Matrix (9) can be represented by:

$$(11) \begin{pmatrix} \bar{F}_0 \\ \bar{F}_L \end{pmatrix} = \begin{pmatrix} \frac{-A}{B} & \frac{1}{B} \\ \frac{-1}{B} & \frac{D}{B} \end{pmatrix} \begin{pmatrix} \bar{\phi} \bar{V}_0 \\ \bar{\phi} \bar{V}_L \end{pmatrix}$$

The solution to matrix (11) is obtained by applying the Inversion Theorem for Laplace transforms:

$$F = \frac{1}{2\pi i} \lim_{\gamma \rightarrow \infty} \int_{\epsilon - i\gamma}^{\epsilon + i\gamma} e^{\lambda t} \bar{F} d\lambda.$$

In order to simplify this calculation, express the transformed flux  $\bar{F}$  as a generalized equation;

$$(12) \quad \bar{F} = \frac{\bar{\phi} M(P)}{B(P)}$$

where  $\bar{\phi}$  is the Laplace transform of the pulse function  $\phi$ ,

$B(P) = \frac{R_t \text{SIN} q x}{q}$ , and  $M(P)$  is either 1, A, or D. Since  $R_t$  is

the total resistivity of the planar construction and  $U = \frac{1}{R_t}$ ,

then  $B(P) = \frac{\text{SIN} q x}{U q}$ .

To find the inverse of equation (12), the Calculus of Residues<sup>2</sup> will be utilized:

1) determine which components in (12) will make the equation undefined --

a. Since  $\bar{\phi}$  is composed of element  $\frac{1}{P}$  (refer equation (5)), if  $P = 0$  then (12) becomes undefined

b. Since (12) is composed of element  $\frac{1}{B(P)}$ , if  $B(P) = 0$  then (12) becomes undefined

2) dealing with the  $\bar{\phi}$  part first --

a. cancel out the element  $\frac{1}{P}$  in  $\bar{\phi}$  and obtain ' $\bar{\phi}'$ ':

$$(13) \left\{ \begin{array}{ll} \bar{\phi}' = \frac{1}{\delta} & , 0 < t \leq \delta \\ \bar{\phi}' = \frac{1}{\delta} - \frac{2e^{-P\delta}}{\delta} & , \delta < t \leq 2\delta \\ \bar{\phi}' = \frac{1}{\delta} - \frac{2e^{-P\delta}}{\delta} + \frac{e^{-2P\delta}}{\delta} & , t > 2\delta \end{array} \right.$$

b. By the Calculus of Residues, part I of the answer is therefore:

$$(14) \lim_{P \rightarrow 0} \frac{d}{dP} \left( \frac{e^{Pt} M(P) \bar{\phi}'}{B(P)} \right) =$$

$$\left[ UM(P) + \frac{U}{\delta} \frac{dM}{dP} - \frac{U^2 M(P)}{\delta} \frac{dB}{dP} \right]_{P=0} , 0 < t \leq \delta$$

$$\left[ \frac{U^2 M(P)}{\delta} \frac{dB}{dP} - \frac{U}{\delta} \frac{dM}{dP} \right]_{P=0} , \delta < t \leq 2\delta$$

$$0, t > 2\delta$$

3) for the  $B(P) = 0$  part --

a. By using the Heaviside Expansion Theorem<sup>3</sup>, part II  
of the answer is:

$$(15) \quad \sum_{r=1}^{RN} \left[ \frac{e^{-W_r t} \bar{\phi} M(W_r)}{W_r} \frac{1}{\frac{dB}{dW}} \right] =$$

$$\sum_{r=1}^{RN} \left[ \frac{e^{-W_r \delta} M(W_r)}{\delta W_r^2} \frac{1}{\frac{dB}{dW}} \right] \quad , t = \delta$$

$$\sum_{r=1}^{RN} \left[ \frac{M(W_r)}{\delta W_r^2} (e^{-2W_r \delta} - 2e^{-W_r \delta}) \frac{1}{\frac{dB}{dW}} \right] \quad , t = 2\delta$$

$$\sum_{r=1}^{RN} \left[ \frac{M(W_r)}{\delta W_r^2} (e^{-W_r \delta j} - 2e^{-W_r \delta (j-1)} + e^{-W_r \delta (j-2)}) \frac{1}{\frac{dB}{dW}} \right] ,$$

$t > 2\delta$ ,  $j = 3, 4, \dots$ ,  $n = \text{number of hours}$

In the above equations, RN is the number of roots of  
 $B(P) = 0$  and  $W_r$  is the negative of each root ( $W_r = -P$ ).

When equations (14) and (15) are combined, when  $\delta$  is  
set to 1 hour, and when M is replaced by D, I, and A, and  
evaluated, the equations for the X, Y, and Z (respectfully)  
response factors are obtained:



The equations for the X response factors are:

$$\begin{aligned}
 x_0 &= u + \left[ u \cdot \left( \frac{dB}{dW} \right)_{W \rightarrow 0.0} \right] - \left[ u^2 \cdot \left( \frac{dB}{dW} \right)_{W \rightarrow 0.0} \right] + \sum_{k=1}^{RN} \left[ \left( \frac{1}{W_k^2} \right) \cdot D_{W_k} \cdot \left( \frac{1}{\frac{dB}{dW}} \right)_{W_k} \cdot e^{-(\delta W_k)} \right] \\
 x_1 &= \left[ u \cdot \left( \frac{dB}{dW} \right)_{W \rightarrow 0.0} \right] + \left[ u^2 \cdot \left( \frac{dB}{dW} \right)_{W \rightarrow 0.0} \right] + \sum_{k=1}^{RN} \left[ \left( \frac{1}{W_k^2} \right) \cdot D_{W_k} \cdot \left( \frac{1}{\frac{dB}{dW}} \right)_{W_k} \cdot (1 - 2e^{-(\delta W_k)}) \cdot e^{-(2\delta W_k)} \right] \\
 x_i &= \sum_{k=1}^{RN} \left[ \left( \frac{1}{W_k^2} \right) \cdot D_{W_k} \cdot \left( \frac{1}{\frac{dB}{dW}} \right)_{W_k} \cdot (1 - e^{-(\delta W_k)})^2 \cdot e^{-[(i+1)\delta W_k]} \right]
 \end{aligned}$$

i=2, 3, 4...NRT

The equations for the Y response factors are:

$$\begin{aligned}
 y_0 &= u - \left[ u^2 \cdot \frac{dB}{dW} \right]_{W \rightarrow 0.0} + \sum_{k=1}^{RN} \left[ \left( \frac{1}{W_k^2} \right) \cdot \left( \frac{1}{\frac{dB}{dW}} \right)_{W_k} \right] \\
 y_1 &= \left[ u^2 \cdot \left( \frac{dB}{dW} \right)_{W \rightarrow 0.0} \right] + \sum_{k=1}^{RN} \left[ \left( \frac{1}{W_k^2} \right) \cdot \left( \frac{1}{\frac{dB}{dW}} \right)_{W_k} \cdot (1 - e^{-(\delta W_k)}) \cdot e^{-(2\delta W_k)} \right] \\
 y_i &= \sum_{k=1}^{RN} \left[ \left( \frac{1}{W_k^2} \right) \cdot \left( \frac{1}{\frac{dB}{dW}} \right)_{W_k} \cdot (1 - e^{-(\delta W_k)})^2 \cdot e^{-[(i+1)\delta W_k]} \right]
 \end{aligned}$$

i=2, 3, 4...NRT

And the equations for the Z response factors are:

$$\begin{aligned}
 z_0 &= u + \left[ u \cdot \left( \frac{dA}{dW} \right)_{W \rightarrow 0.0} \right] - \left[ u^2 \cdot \left( \frac{dB}{dW} \right)_{W \rightarrow 0.0} \right] + \sum_{k=1}^{RN} \left[ \left( \frac{1}{W_k^2} \right) \cdot A_{W_k} \cdot \left( \frac{1}{\frac{dB}{dW}} \right)_{W_k} \cdot e^{-(\delta W_k)} \right] \\
 z_1 &= \left[ u \cdot \frac{dA}{dW} \right]_{W \rightarrow 0.0} + \left[ u^2 \cdot \frac{dB}{dW} \right]_{W \rightarrow 0.0} + \sum_{k=1}^{RN} \left[ \left( \frac{1}{W_k^2} \right) \cdot A_{W_k} \cdot \left( \frac{1}{\frac{dB}{dW}} \right)_{W_k} \cdot (1 - 2e^{-(\delta W_k)}) \cdot e^{-(\delta W_k)} \right] \\
 z_i &= \sum_{k=1}^{RN} \left[ \left( \frac{1}{W_k^2} \right) \cdot A_{W_k} \cdot \left( \frac{1}{\frac{dB}{dW}} \right)_{W_k} \cdot (1 - e^{-(\delta W_k)})^2 \cdot e^{-[(i+1)\delta W_k]} \right]
 \end{aligned}$$

i=2, 3, 4...NRT

### Conduction Matrix Element B

To begin with, define a purely arbitrary wall composed of eight inch thick concrete block (outside surface), air space, gypsum board, and inside air. Using the material characteristics described in APPENDIX A, the equation for the overall conduction matrix element B will be obtained.

The equation for B is determined by the following matrix algebra:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} * \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} * \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix} * \begin{pmatrix} a_4 & b_4 \\ c_4 & d_4 \end{pmatrix}$$

$$\Rightarrow B = (a_1 * a_2 + b_1 * c_2) * (a_3 * b_4 + b_3 * d_4) + \\ (a_1 * b_2 + b_1 * d_2) * (c_3 * b_4 + a_3 * d_4)$$

As generalized equations, each layer's matrix values are determined by:

#### NON-AIR LAYER

for  $W > 0.0$ :

$$a_i = \cos(\sqrt{W} * BETA_i)$$

$$b_i = R_i * \sin(\sqrt{W} * BETA_i) / (\sqrt{W} * BETA_i)$$

$$c_i = -(\sqrt{W} * BETA_i) * \sin(\sqrt{W} * BETA_i) / R_i$$

$$d_i = a_i$$

$$da_i = \frac{BETA_i * \sin(\sqrt{W} * BETA_i)}{2.0 * \sqrt{W}}$$

$$db_i = \left( \frac{R_i}{2.0 * W} \right) * \left( \frac{\sin(\sqrt{W} * BETA_i)}{\sqrt{W} * BETA_i} - \cos(\sqrt{W} * BETA_i) \right)$$

$$dc_i = \left( \frac{BETA_i}{2 * R_i} \right) * \left( \frac{\sin(\sqrt{W} * BETA_i)}{\sqrt{W}} + BETA_i * \cos(\sqrt{W} * BETA_i) \right)$$

$$dd_i = da_i$$

as  $W \rightarrow 0.0$

$$A = 1.0$$

$$B = R_t$$

$$C = 0.0$$

$$D = 1.0$$

$$da_i = BETA_i * BETA_i / 2.0$$

$$db_i = R_i * BETA_i * BETA_i / 6.0$$

$$dc_i = BETA_i * BETA_i / R_i$$

$$dd_i = da_i$$

AIR LAYER

$$a_i = 1.0$$

$$b_i = R_i$$

$$c_i = 0.0$$

$$d_i = a_i$$

Upon substituting the appropriate expressions for the a's, b's, c's, and d's (keeping in mind that the second and fourth layers are air), the B equation then becomes:

$$\begin{aligned} B = & \left[ \cos(\sqrt{W} * \text{BETA}_1) * \left[ R_4 * \cos(\sqrt{W} * \text{BETA}_3) + \right. \right. \\ & \left. \left. R_3 * (\sqrt{W} * \text{BETA}_3)^{-1} * \sin(\sqrt{W} * \text{BETA}_3) \right] \right] + \\ & \left[ \left[ R_2 * \cos(\sqrt{W} * \text{BETA}_1) + \right. \right. \\ & \left. \left. R_1 * (\sqrt{W} * \text{BETA}_1)^{-1} * \sin(\sqrt{W} * \text{BETA}_1) \right] * \right. \\ & \left. \left[ -R_4 * (R_3)^{-1} * \sqrt{W} * \text{BETA}_3 * \sin(\sqrt{W} * \text{BETA}_3) + \right. \right. \\ & \left. \left. \cos(\sqrt{W} * \text{BETA}_3) \right] \right] \end{aligned}$$

The only unknown value in the B equation is the variable W. When  $0.0 < W < \infty$ , B graphs out as:

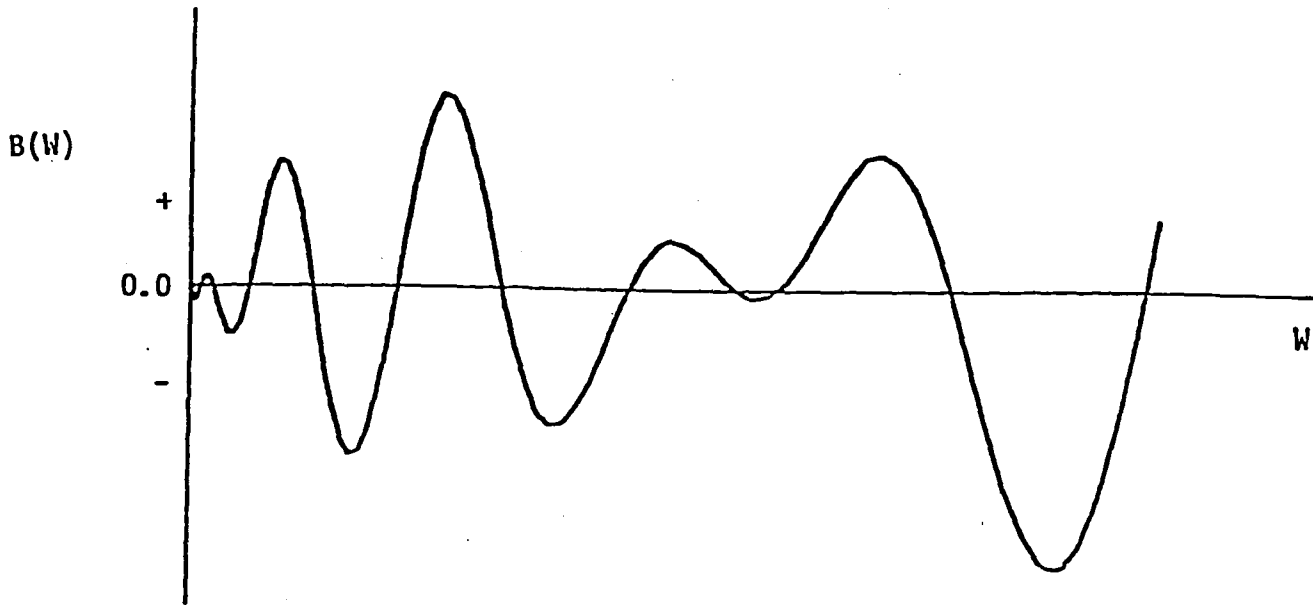


FIGURE 2

In order to calculate the thermal response factors (X, Y, and Z) for this planar construction, the values of W which makes  $B(W)=0.0$  must be determined. As can be seen in Figure 2, this complicated wave has no predictable period. Therefore, a mathematical root finding technique must be used to solve  $B(W)=0.0$ .

CALCULATING ROOTS  
Original Root Procedure

The calculation sequence for this method is:

1. Calcualte first root  $>30.0$  using sequence 3;  
and set  $ROOT(1) =$  to this root; set  $LAST = 1$
2. Determine root intervals for locating all roots  
 $<30.0$  but  $>0.0$ 
  - a) if  $LAST = 1$  then set  $W1 = 0.0001$   
else  $W1 = ROOT(LAST - 1)$
  - b)  $W2 = ROOT(LAST)$
  - c)  $W2 = W2 - 0.0001$   
 $W1 = W1 + 0.0001$
  - d)  $DELTA = (W2 - W1)/20.0$   
 $J = 1$   
 $IC = 0$
  - e)  $W3 = W1 + (J*DELTA)$
  - f) calculate  $B(W1)$  and the derivative of  $B$ ,  $DB(W1)$
  - g) calculate  $B(W3)$  and the derivative of  $B$ ,  $DB(W3)$
  - h) if  $B(W1) * B(W3) < 0.0$  then have a root interval  
so set  $W2 = W3$  and go to 3
  - i) if  $DB(W1) * DB(W3) < 0.0$ , set  $IC = IC + 1$
  - j) if  $IC - 2 \geq 0.0$  then have a root interval so set  
 $W2 = W3$  and go to 3
  - k)  $J = J + 1$
  - l) if  $J > 19$  then no root interval so set  $LAST = LAST - 1$   
and go to 4
  - m)  $B(W1) = B(W3)$   
 $DB(W1) = DB(W3)$   
 $W3 = W1 + (J*DELTA)$   
go to 2g

3. Determine the root that is in the interval (W1, W2)

- a)  $N = 1$   
 $J = 1$   
 $BP3 = W1$
- b) calculate  $B(W1)$   
 $B2 = B(W1)$
- c)  $DELTA = (W2 - W1) / (20.0 * N)$
- d)  $W3 = W1 + (J * DELTA)$
- e) if  $W3 - W2 > 0.0$  go to 3i
- f)  $N = N + 1$
- g) if  $N > 25$  then there is no root  $< 30.0$  so go to 4
- h) set  $J = 1$  and go to 3c
- i) calculate  $B(W3)$   
 $B3 = B(W3)$
- j) if  $B(W1) * B(W3) < 0.0$  go to 3l
- k)  $J = J + 1$   
 $B2 = B3$   
 $BP3 = W3$   
go to 3d
- l)  $A = BP3$   
 $B = W3$   
 $FA = B2$   
 $FB = B3$
- m) perform Bisection Method on interval (A, B) until root is located (i.e.  $ABS(B(A) - B(B)) \leq 1.0E-8$ ), go to 4

4.  $LAST = LAST + 1$   
 $ROOT(LAST) = ROOT(LAST - 1)$   
 $ROOT(LAST - 1) = W3$
5. Go to 2
6. End

Although the Bisection Method guarantees a root once the root interval is found, it requires a significant amount of execution time to actually locate the root. Compounded with the time needed to locate each root interval, this method is quite inefficient.



## New Root Procedure

The basis for reducing execution time and cost lies in two categories: originating a faster method to locate the root intervals and using a more efficient algorithm for determining the roots.

### Root Intervals:

As was shown, the temperature flux matrix element B is mainly composed of sines and cosines. Taken individually, each non-air layer's sine and cosine equations have calculatable roots. Therefore, this result will be used to segment the B wave (Figure 2) into possible root intervals.

For each non-air layer, use its BETA value to determine the segmenter roots (i.e.  $\sqrt{N}$ ).

$$RVC = (i-0.5) * \pi / BETA \quad (\text{for cosine})$$

$$RVS = i * \pi / BETA \quad (\text{for sine})$$

$$\text{for } i = 1, 2, 3, 4, \dots$$

Upon doing this for each non-air layer and then sorting all of the RVC and RVS values in ascending order, an array V is obtained that can be used for root interval checking.

$$V \equiv (V_1, V_2, V_3, \dots), V_1 < V_2 < V_3 \dots$$

where  $\{V_1, V_2\}$  is root interval 1

$\{V_2, V_3\}$  is root interval 2

etc.

### Determining the Roots of B :

Having determined the root interval checking array, the actual roots can then be found.

The calculation sequence can be summerized as:

- a) set  $i=1$
- b) set  $G1=V_i$
- c) set  $G2=V_{i+1}$
- d) calculate  $B(G1)$
- e) calculate  $B(G2)$
- f) if  $ABS(B(G1))$  or  $ABS(B(G2)) \leq 1.0E-8$ , a root has been found so go to h.
- g) else check the possible root interval at one-tenth segments:
  - 1) set  $D = G2-G1$ ,  $FD = 0.0$
  - 2)  $FK = FK+1.0$ , if  $FK > 10.0$  then the entire interval has been checked so increment  $i$  and go to b
  - 3) else calculate segment end point:  
 $GNEW = PINT_i + (D*FK/10.0)$
  - 4) calculate  $FNEW = B(GNEW)$
  - 5) check sign of  $FNEW$  against sign of  $B(G1)$ :  
 if  $FNEW*B(G1) > 0.0$  then no root in this segment, so reset segment front point,  $G1 = GNEW$  and  $B(G1) = FNEW$  then go to g2

- 6) else there is a root in this segment; store segment front point values:  $G_X = G_1$  and  $F_X = B(G_1)$ , go to h
- h) attempt to locate the root by first using the Secant method; if the root 'guess' ever exceeds the end points of the segment being checked, then this method fails so use the more time consuming Bisection method on the segment end points (i.e.  $G_X$  and  $G_{NEW}$ ) to locate the root; go to i
- i) the root value obtained was actually the square root of the root value so compute the actual root (i.e.  $ROOT = \text{root value} * \text{root value}$ ); if  $ROOT > 30.0$  go to k
- j) reset segment front point:  $G_1 = G_{NEW}$  and  $B(G_1) = F_{NEW}$ ; go to g2 to check next segment in this interval
- k) end

## VALIDATION OF NEW ROOT PROCEDURE

Ten test cases were run to compare the original RESFAC against the new program. Input requirements and output for both programs are in APPENDIX A. Planar construction ranged from a simple, two material layer wall to a complex, ten material layer roof. A wide sampling was used for comparison testing and execution timing. The data input and both program's output for the test cases are given in APPENDIX B.

All testing was done on a high speed, Control Data Corp. CYBER 173 computer. Both programs use single precision FORTRAN code and were run on a time sharing, batch mode. TABLE I lists the execution times and costs for the test cases.

TABLE I

## EXECUTION TIME &amp; COST COMPARISON\*

TEST CASE	EXECUTION** TIME		% REDUCTION	EXECUTION*** COST		% REDUCTION
	OLD	NEW		OLD	NEW	
1	0.965	0.228	76.4	0.57	0.45	21.0
2	5.642	0.131	97.7	0.61	0.45	26.2
3	1.712	0.292	82.9	0.58	0.45	22.4
4	3.566	0.415	88.4	0.60	0.45	25.0
5	2.181	0.344	84.2	0.58	0.45	22.4
6	17.584	1.341	92.4	0.74	0.46	37.8
7	6.767	0.696	89.7	0.63	0.45	28.6
8	5.136	0.533	89.6	0.61	0.45	26.2
9	5.618	0.601	89.3	0.62	0.45	27.4
10	17.314	1.034	94.0	0.73	0.46	40.0

Average reduction in execution time was 88.5%

Average reduction in execution cost was 27.7%

\* All runs were made on a Control Data Corp. CYBER-173

\*\* Execution time is in Central Processor (CP) seconds

\*\*\* Execution cost is in System Resource Units (SRU)

## CONCLUDING REMARKS

From the analysis performed, it was found that thermal response factors are time dependent heat transfer coefficients whose derivations require complex mathematics. In deriving the response factor equations, it was necessary to calculate the Laplace transforms and the inverse Laplace transforms for the heat conduction equation and the heat flux equation - both of which are acted upon by a pulse function comprised of the Heaviside unit function. Furthermore, the keys to obtaining the inverses were found to be the Calculus of Residues Theorem and the Heaviside Expansion Theorem.

Testing of the original Response Factor Program revealed that its computational time requirement was not exorbitant. Thus, no basis was found for the criticism expounded against it. However, an analysis of the program code did reveal inefficiencies in the root calculating procedure for conduction matrix element B. This warranted the development of a new root calculating procedure.

From an analysis of the response factor derivation, it was decided that each material layers SIN and COSINE components could be utilized in an effective manner. Since these components are periodic functions, their zero roots are easily calculated. Once these component roots are obtained for each layer and then ascendingly sorted, a root interval checking array can be obtained. Segments from this array (values at array positions  $i$  and  $i+1$ ) can then be

sectionalized into ten parts and each of these mini-segments can be checked to determine if it contains a root for matrix element  $B = 0$ . This approach to a new root calculating procedure proved to be very appropriate.

In comparison testing, the new Response Factor Program executed more efficiently and more accurately than the original RESFAC. The new program's computational time requirement was much less than that of the original program - a substantial 88.5% average reduction.

NOTES



## APPENDIXES

## APPENDIX A

### PROGRAM INPUT/OUTPUT

Input to the Response Factor Program (FTN 4.8 extended)

consists of:

- a) number of material layers
- b) for each material layer (arranged-from outside to inside), its:
  - i) thickness (FT); if air layer then input 0.0
  - ii) thermal conductivity (BTU/HR.FT.F), if air layer then input 0.0
  - iii) density (LB/FT<sup>3</sup>), if air layer then input 0.0
  - iv) specific heat (BTU/LB.F); if air layer then input 0.0
  - v) thermal resistance (HR.FT<sup>2</sup>.F/BTU); if NON air layer then input 0.0
  - vi) alpha-numeric description (30 characters)

Output from the Response Factor Program consists of:

- a) echo of the input
- b) thermal conductance of planar construction
- c) response factors series X, Y, and Z
- d) number of hours to reach common ration
- e) number of response factors per set
- f) common ratio

The common ratio is defined as the common rate at which the response factors tend to decrease:

$$\frac{X_{i+1}}{X_i} = \frac{Y_{i+1}}{Y_i} = \frac{Z_{i+1}}{Z_i}$$

APPENDIX B  
TEST CASES



TABLE 1 (Cont.)

ROOTS		
ROOT #	OLD	NEW
1	0.32755449581283	0.32755450010252
2	1.93392856508856	1.93392855643586
3	5.10625948098138	5.10625947586985
4	9.86100164604460	9.86100166757376
5	16.19980403179670	16.19980403077625
6	24.12301443118963	24.12301442902663
7	33.63074066210538	33.63074065640694
B (ROOT)		
FOR ROOTS #	OLD	NEW
1	1.191627E-8	-8.96300E-11
2	7.966970E-9	8.20000E-13
3	-2.796240E-9	4.00000E-14
4	-1.886340E-9	6.49869E-9
5	-3.086200E-9	5.00000E-14
6	5.348400E-10	-7.00000E-14
7	1.191860E-9	1.00000E-15

TABLE 2

DESCRIPTION OF CONSTRUCTION						
LAYER NUMBER	THICKNESS FT	CONDUCTIVITY BTU PER (HR)(FT)(F)	DENSITY LB PER CU FT	SPECIFIC HEAT BTU PER (LB)(F)	RESISTANCE (HR)(SQ FT)(F) PER BTU	
1	.0050	26.000	480.0	.1000	0.0000	SHEET METAL
2	0.0000	0.000	0.0	0.0000	.6893	INSIDE AIR

## OLD RESFAC

RESPONSE FACTORS			
HOURS	X	Y	Z
0	1.6987382517	1.4587719164	1.4588055812
1	-.2399326768	.0000336584	-.0000000063
2	0.0000000000	0.0000000000	-.0000000000
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 2			
NUMBER OF RESPONSE FACTORS PER SET = 3			
COMMON RATIO = 0.0000000000			

## NEW RESFAC

RESPONSE FACTORS			
HOURS	X	Y	Z
0	1.6987382517	1.4587719164	1.4588055812
1	-.2399326768	.0000336584	-.0000000063
2	0.0000000000	0.0000000000	-.0000000000
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 2			
NUMBER OF RESPONSE FACTORS PER SET = 3			
COMMON RATIO = 0.0000000000			

## ROOTS

OLD	NEW
"NO ROOT IN FALSE"	"NO ROOT <30.0"

TABLE 3

DESCRIPTION OF CONSTRUCTION						
LAYER NUMBER	THICKNESS (FT)	CONDUCTIVITY (BTU PER (INP)(FT)(F))	DENSITY (LB PER CU FT)	SPECIFIC HEAT (BTU PER (LB)(F))	RESISTANCE (HR)(SQ FT)(F) PER BTU	
1	.0833	.400	116.0	.2000	0.0060	STUCCO
2	.6670	.600	61.0	.2000	0.0000	CONCRETE BLOCK
3	0.0000	0.000	0.0	0.0000	.6850	INSIDE AIR

## OLD RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	3.3351989142	.0047349047	1.0268924803
1	-2.0357518186	.0774691384	-.2325472853
2	-.3181873898	.1220914743	-.1034496668
3	-.1673243788	.0956263776	-.0644821910
4	-.1057356979	.0658090798	-.0423797458
5	-.0697646918	.0441981869	-.0281976595
6	-.0464561062	.0295390350	-.0188093241
7	-.0309918423	.0197221399	-.0125533739
8	-.0206860498	.0131650601	-.0083790413
9	-.0134074819	.0097876637	-.0055929090
10	-.0092163408	.0058657058	-.0037332161
11	-.0061518258	.0039153118	-.0024918902
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 11			
NUMBER OF RESPONSE FACTORS PER SET = 12			
COMMON RATIO = .6674017264			

## NEW RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	3.3351989055	.0047349117	1.0268924801
1	-2.0357518095	.0774691303	-.2325472859
2	-.3181873891	.1220914743	-.1034496663
3	-.1673243787	.0956263777	-.0644821907
4	-.1057356980	.0658090799	-.0423797457
5	-.0697646920	.0441981870	-.0281976594
6	-.0464561064	.0295390352	-.0188093241
7	-.0309918425	.0197221400	-.0125533739
8	-.0206860499	.0131650602	-.0083790413
9	-.0134074820	.0097876638	-.0055929090
10	-.0092163409	.0058657058	-.0037332161
11	-.0061518259	.0039153119	-.0024918903
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 11			
NUMBER OF RESPONSE FACTORS PER SET = 12			
COMMON RATIO = .6674017214			

TABLE 3 (Cont.)

## ROOTS

ROOT #	OLD	NEW
1	0.40422829270580	0.40422829118816
2	1.98888907867067	1.98888908214629
3	4.95990390912746	4.95990391749302
4	9.33689576656178	9.33689576507368
5	15.23955713347846	15.23955713476266
6	22.76863355467356	22.76863355381067
7	31.86645567696542	31.86645567723178

## B (ROOTS)

FOR ROOTS #	OLD	NEW
1	-4.60508E-9	-2.500E-13
2	-3.28262E-9	-7.000E-14
3	4.50591E-9	-7.000E-14
4	5.74390E-10	-3.000E-14
5	3.98570E-10	-1.100E-13
6	2.42400E-10	1.857E-11
7	5.67000E-11	-7.600E-13



TABLE 4

DESCRIPTION OF CONSTRUCTION							
LAYER NUMBER	THICKNESS FT	CONDUCTIVITY BTU PER (HR)(FT)(F)	DENSITY LB PER CU FT	SPECIFIC HEAT BTU PER (LB)(F)	RESISTANCE (HR)(SQ FT)(F) PER BTU		
1	1.0000	.470	61.0	.2000	0.0000	CONCRETE	
2	0.0000	0.000	0.0	0.0000	.9100	AIR SPACE	
3	.0625	.420	100.0	.2000	0.0000	GYPSUM BOARD	
4	0.0000	0.000	0.0	0.0000	.6850	INSIDE AIR	

## OLD RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	2.7019910060	.0000045494	.9028094952
1	-1.5827901353	.0013717141	-.3592850806
2	-.2604623224	.0104498496	-.0959141188
3	-.1353416110	.0217968584	-.0425646423
4	-.0878924779	.0269294928	-.0267846451
5	-.0643224585	.0269628958	-.0198978947
6	-.0504462651	.0246772257	-.0158734504
7	-.0411015290	.0216900394	-.0130731950
8	-.0341619995	.0187069269	-.0109294008
9	-.0286814781	.0159940218	-.0092032465
10	-.0242006794	.0136146009	-.0077768730
11	-.0204699917	.0115652090	-.0065827591
12	-.0173351451	.0096144694	-.0055766122
13	-.0146889411	.0083247083	-.0047261531
14	-.0124502106	.0070554147	-.0040061772
15	-.0105541392	.0059857498	-.0033962047
16	-.0089474249	.0050750956	-.0028792386
17	-.0075855568	.0043028691	-.0024410192
18	-.0064310775	.0036480965	-.0020695194
19	-.0054523455	.0030929417	-.0017545676
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 19			
NUMBER OF RESPONSE FACTORS PER SET = 20			
COMMON RATIO = .8478183024			

TABLE 4 (Cont.)

## NEW RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	2.7019910343	.0000045273	.9028094952
1	-1.5427901625	.0013717364	-.3592850790
2	-.2604623235	.0104498499	-.0959141192
3	-.1353416117	.0217968287	-.0425646427
4	-.0979924743	.0269294930	-.0267846454
5	-.0443224589	.0269628558	-.0198979049
6	-.0504462653	.0246772257	-.0158734506
7	-.0411015296	.0216900394	-.0130731952
8	-.0341619995	.0187089269	-.0109294008
9	-.0285814741	.0159940218	-.0092032465
10	-.0242006794	.0136146008	-.0077768730
11	-.0204699916	.0115652089	-.0063827591
12	-.0173351450	.0098144694	-.0055766122
13	-.0146889410	.0083747082	-.0047261531
14	-.0124627104	.0070594146	-.0040061772
15	-.0105561391	.0059657497	-.0033962047
16	-.0089474248	.0050750955	-.0028792385
17	-.0075855557	.0043028690	-.0024410192
18	-.0064310774	.0036480965	-.0020695194
19	-.0054523455	.0030929417	-.0017545676
NUMBER OF HOURS REQUIRED TO REACH COMMON PATIO = 10			
NUMBER OF RESPONSE FACTORS PER SET = 20			
COMMON PATIO = .8478183011			

TABLE 4 (Cont.)

ROOTS		
ROOT #	OLD	NEW
1	0.16508893214859	0.16508893366130
2	0.88663631632084	0.88663631571600
3	1.82111532969845	1.82111532712176
4	2.70897699922766	2.70897699132630
5	4.87369736475012	4.87369737239763
6	7.89216455071565	7.89216455605310
7	11.68507789151147	11.68507788855550
8	16.24256497751424	16.24256497291390
9	21.56196345126659	21.56196345365345
10	27.64206980186293	27.64206980859319
11	34.48177223186940	34.48177223524772
B(ROOTS)		
FOR ROOTS #	OLD	NEW
1	2.128693E-8	-3.80000E-13
2	1.763430E-9	7.00000E-14
3	-3.889470E-9	-6.41180E-10
4	1.059484E-8	-2.67600E-11
5	2.079299E-8	7.00000E-14
6	-1.933347E-8	-1.03077E-9
7	-1.325572E-8	-2.71100E-11
8	2.175127E-8	7.33800E-11
9	1.086137E-8	-1.09000E-12
10	-2.711985E-8	-1.61000E-12
11	1.036738E-8	-4.57000E-10

TABLE 5

DESCRIPTION OF CONSTRUCTION							
LAYER NUMBER	THICKNESS FT	CONDUCTIVITY BTU PER (HR)(FT)(F)	DENSITY LB PER CU FT	SPECIFIC HEAT BTU PER (LB)(F)	RESISTANCE (HR)(SQ FT)(F) PER BTU		
1	.0417	.830	55.0	.4000	0.0000	STONE	
2	.0313	.110	70.0	.4000	0.0000	FELT	
3	.1670	.025	2.0	.2000	0.0000	CELLULAR GLASS	
4	.0050	26.000	480.0	.1000	0.0000	SHEET METAL	
5	0.0000	0.000	0.0	0.0000	.6950	INSIDE AIR	

## OLD RESFAC

RESPONSE FACTORS				
HOUR	X	Y	Z	
0	1.8974134526	.0743183191	.3484881084	
1	-1.7660938687	.0549331620	-.2179800359	
2	-.0014456694	.0006165926	-.0006354967	
3	-.0000034153	.0000024075	-.0000020801	
4	-.0000000105	.0000000084	-.0000000070	
5	-.0000000000	.0000000000	-.0000000000	
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 5				
NUMBER OF RESPONSE FACTORS PER SET = 6				
COMMON RATIO = .0033832861				

## NEW RESFAC

RESPONSE FACTORS				
HOUR	X	Y	Z	
0	1.8974134525	.0743183181	.3484881084	
1	-1.7660938686	.0549331620	-.2179800359	
2	-.0014456694	.0006165926	-.0006354967	
3	-.0000034153	.0000024075	-.0000020801	
4	-.0000000105	.0000000084	-.0000000070	
5	-.0000000000	.0000000000	-.0000000000	
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 5				
NUMBER OF RESPONSE FACTORS PER SET = 6				
COMMON RATIO = .0033832861				

TABLE 5 (Cont.)

ROOTS			
ROOT #	OLD	NEW	
1	5.68890723428157	5.68890723527994	
2	7.23930079653397	7.23930076296122	
3	23.82487390640392	23.82487391298810	
4	46.56416592840105	46.56416592806750	
B (ROOTS)			
FOR ROOTS #	OLD	NEW	
1	1.4505E-9	4.42000E-12	
2	4.4640E-9	-3.36164E-9	
3	9.4499E-9	-9.11880E-10	
4	1.0243E-9	6.80000E-13	

TABLE 6

DESCRIPTION OF CONSTRUCTION						
LAYER NUMBER	THICKNESS FT	CONDUCTIVITY BTU PER (HP)(FT)(F)	DENSITY LB PER CU FT	SPECIFIC HEAT BTU PER (LB)(F)	RESISTANCE (HP)(SQ FT)(F) PER BTU	
1	.0400	.400	116.0	.2000	0.0000	STUCCO
2	.0430	.015	74.0	.2000	0.0000	INSULATION
3	.0400	.400	116.0	.2000	0.0000	STUCCO
4	1.0000	.340	106.0	.2000	0.0000	MASONARY
5	.1250	.250	5.7	.2000	0.0000	RIGID INSULATION
6	.1250	.420	100.0	.2000	0.0000	GYP SUM
7	0.0000	0.000	0.0	0.0000	.6850	INSIDE AIR

## OLD RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	1.4133452868	-.0600016845	1.0234028097
1	-1.1675788059	.0000016986	-.2922661790
2	-.0267241362	.0606006888	-.1269399552
3	-.0094665212	.0000158459	-.0696275927
4	-.0058867493	.0000465331	-.0445554251
5	-.0045961430	.0002893197	-.0318517154
6	-.0038760191	.0005614340	-.0245613444
7	-.0033688850	.0009223559	-.0199316112
8	-.0030284883	.0012630118	-.0167516158
9	-.0027478187	.0015717229	-.0144353344
10	-.0025216815	.0018335561	-.0126736599
11	-.0023349639	.0020448307	-.0112905141
12	-.0021778771	.0022080835	-.0101785182
13	-.0020437032	.0023286877	-.0092681270
14	-.0019276198	.0024128978	-.0085119026
15	-.0018260430	.0024668386	-.0078760512
16	-.0017362357	.0024960470	-.0073356433
17	-.0016560803	.0025053151	-.0068717849
18	-.0015936905	.0024986835	-.0064698635
19	-.0015183328	.0024795039	-.0061164099
20	-.0014563340	.0024505270	-.0058083259
21	-.0014030280	.0024139934	-.0055323389
22	-.0013517122	.0023717167	-.0052866038
23	-.0013038151	.0023251678	-.0050604034
24	-.0012588698	.0022755169	-.0048559191
25	-.0012164939	.0022237162	-.0046680514
26	-.0011763732	.0021705143	-.0044942787
27	-.0011382486	.0021165137	-.0043325451
28	-.0011019055	.0020621967	-.0041811707
29	-.0010671655	.0020079211	-.0040367794
30	-.0010338796	.0019539995	-.0039042409
31	-.0010019227	.0019006553	-.0037766241
32	-.0009711891	.0018480652	-.0036515995
33	-.0009415890	.0017963638	-.0035342082
34	-.0009130455	.0017456515	-.0034282382
35	-.0008854922	.0016960017	-.0033218035
36	-.0008586716	.0016474659	-.0032195287
37	-.0008331331	.0016000783	-.0031210956
38	-.0008082321	.0015538592	-.0030262333
39	-.0007841290	.0015088160	-.0029347082
40	-.0007607860	.0014649593	-.0028463187
41	-.0007381769	.0014222651	-.0027608689

TABLE 6 (Cont.)

42	-.0007162661	.0013807359	-.0026782639
43	-.0006950287	.0013403523	-.0025983063
44	-.0006744394	.0013010956	-.0025208934
45	-.0006544751	.0012629448	-.0024459142
46	-.0006351140	.0012258772	-.0023732683
47	-.0006163353	.0011898689	-.0023028634
48	-.0005981199	.0011548952	-.0022346145
49	-.0005804492	.0011209307	-.0021684428
50	-.0005633057	.0010879499	-.0021042747
51	-.0005466728	.0010559273	-.0020402011
52	-.0005305344	.0010248373	-.0019816770
53	-.0005149752	.0009946547	-.0019231205
54	-.0004996964	.0009653545	-.0018663132

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 54

NUMBER OF RESPONSE FACTORS PER SET = 55

COMMON RATIO = .9703098791

## NEW RESFAC

RESPONSE FACTORS				
HOUR	X	Y	Z	
0	1.4133454204	.0000000000	1.0234016119	
1	-1.1675789617	.0000000016	-.7922650222	
2	-.0287241355	.0000000684	-.1269392500	
3	-.0094645109	.0000158453	-.0696275884	
4	-.0058867490	.0000965324	-.0445554213	
5	-.0045961418	.0002893190	-.0318517119	
6	-.0038760181	.0005814334	-.0245613414	
7	-.0031888841	.0009223553	-.0199316084	
8	-.0030284874	.0012630113	-.0167516133	
9	-.0027478179	.0015717225	-.0144353321	
10	-.0025214907	.0018335557	-.0128736568	
11	-.0023349631	.0020448303	-.0112905121	
12	-.0021778743	.0022080832	-.0101785164	
13	-.0020437025	.0023286874	-.0092681253	
14	-.0019278171	.0024178975	-.0085119010	
15	-.0018260424	.0024868383	-.0078760497	
16	-.0017342780	.0024960468	-.0073354418	
17	-.0016560797	.0025053149	-.0068717835	
18	-.0015838500	.0024986833	-.0064698622	
19	-.0015183322	.0024795038	-.0061184088	
20	-.0014583335	.0024505269	-.0058043248	
21	-.0014038275	.0024139934	-.0055323378	
22	-.0013517118	.0023717186	-.0052846028	
23	-.0013038144	.0023251678	-.0050604025	
24	-.0012588493	.0022755189	-.0048559183	
25	-.0012164235	.0022237163	-.0046680506	
26	-.0011763728	.0021705143	-.0044962779	
27	-.0011382682	.0021165138	-.0043325444	
28	-.0011019051	.0020621908	-.0041811700	
29	-.0010671651	.0020079212	-.0040387788	
30	-.0010338733	.0019539906	-.0039042403	
31	-.0010019224	.0019006554	-.0037746238	
32	-.0009711888	.0018480453	-.0036551890	
33	-.0009415887	.0017963639	-.0035392077	
34	-.0009130452	.0017456517	-.0034282378	
35	-.0008855919	.0016960019	-.0033218031	
36	-.0008588713	.0016474661	-.0032195284	
37	-.0008331328	.0016000785	-.0031210955	
38	-.00080802319	.0015538594	-.0030282330	
39	-.0007841288	.0015088182	-.0029347079	
40	-.0007607878	.0014649555	-.0028453185	
41	-.00073881747	.0014222653	-.0027608887	

TABLE 6 (Cont.)

42	-.0007162659	.0013907361	-.0026782637
43	-.0004950285	.0013403525	-.0025983062
44	-.0005744793	.0013010958	-.0025208932
45	-.0006544750	.0012629450	-.0024459141
46	-.0006351138	.0012258774	-.0023732682
47	-.0006163357	.001198691	-.0023028633
48	-.0005981198	.0011548554	-.0022346144
49	-.0005804491	.0011209309	-.0021654428
50	-.0005633056	.0010879501	-.0021042747
51	-.0005466727	.0010559275	-.0020420411
52	-.0005305343	.0010248375	-.0019816770
53	-.0005148751	.0009946549	-.0019231205
54	-.0004996901	.0009653547	-.0018663132
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 54			
NUMBER OF RESPONSE FACTORS PER SET = 55			
COMMON RATIO = .9705098860			

## ROOTS

ROOT #	OLD	NEW
1	0.02993369702080	0.02993368988454
2	0.21242985580688	0.21242984890035
3	0.61496372900272	0.61496373599719
4	1.09618246581550	1.09618246155674
5	1.66638739781747	1.66638739147555
6	2.32695013035205	2.32695012724621
7	2.96406703559819	2.96406703639735
8	4.26934120771537	4.26934121302924
9	5.96470442943337	5.96470443677839
10	7.90782511376565	7.90782511203173
11	9.38101582073091	9.38101581368824
12	10.74572406709456	10.74572406078107
13	13.20137627111041	13.20137627948367
14	15.14241773577010	15.14241774014755
15	16.58063080041711	16.58063079648036
16	19.19008477162163	19.19008477625005
17	20.55331112918213	20.55331113165494
18	23.66926094969699	23.66926094966527
19	27.32398281315591	27.32398280793188
20	29.01735456492725	29.01735456421909
21	32.16377783473581	32.16377783283110



TABLE 6 (Cont.)

B(ROOTS)		
FOR ROOTS #	OLD	NEW
1	-1.46805208E-6	1.00000E-14
2	4.47713380E-7	-9.00000E-14
3	1.88247280E-7	-1.40000E-13
4	5.84924700E-8	2.24335E-9
5	-5.49342000E-8	-1.27855E-9
6	2.11869900E-8	-1.40000E-13
7	7.39351000E-9	3.80000E-13
8	-9.94726500E-8	-1.34000E-12
9	1.35020560E-7	-3.19200E-10
10	1.47768100E-8	4.58000E-12
11	-3.02108600E-8	7.20000E-13
12	2.44352100E-8	-4.60000E-13
13	2.34856100E-8	-2.11340E-10
14	-5.98533000E-9	8.00000E-14
15	-4.85016000E-9	7.00000E-13
16	-4.95923000E-9	1.48200E-11
17	3.20458000E-9	1.74200E-10
18	2.62937000E-9	2.54531E-92
19	-8.93764000E-9	-8.12000E-12
20	1.14600000E-9	-8.40000E-12
21	-6.36227000E-9	-2.40000E-13

TABLE 7

DESCRIPTION OF CONSTRUCTION						
LAYER NUMBER	THICKNESS FT	CONDUCTIVITY BTU PER (HR)(FT)(F)	DENSITY LB PER CU FT	SPECIFIC HEAT BTU PER (LB)(F)	RESISTANCE (HR)(SQ FT)(F) PER BTU	
1	.3330	.770	125.0	.2200	0.0000	FACE BRICK
2	0.0000	0.000	0.0	0.0000	.9100	AIR SPACE
3	.0833	.070	37.0	.6000	0.0000	SHEATHING BOARD
4	.3330	.025	5.7	.2000	0.0000	INSULATION
5	.0625	.420	100.0	.2000	0.0000	GYPSUM BOARD
6	0.0000	0.000	0.0	0.0000	.6850	INSIDE AIR

## OLD RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	5.1759961919	.0000062162	.8567490859
1	-3.2693245014	.0001735606	-.5063325817
2	-.7627360815	.0019875908	-.1769113994
3	-.3954482090	.0053818003	-.0661057275
4	-.2236660860	.0077112856	-.0258009737
5	-.1372336219	.0082363141	-.0166121783
6	-.0902026585	.0075773934	-.0047087431
7	-.0624053982	.0064230232	-.0023146240
8	-.0446903989	.0051940987	-.0012785386
9	-.0327070454	.0040653314	-.0007867634
10	-.0242524535	.0031605263	-.0005255624
11	-.0181221793	.0024210054	-.00033703670
12	-.0136018523	.0018436046	-.0002693425
13	-.0102351670	.0013969911	-.0001993224
14	-.0077130398	.0010593995	-.0001488973
15	-.0058172472	.0008012546	-.0001117858
16	-.0043895017	.0006055852	-.0000841465
17	-.0033130658	.0004575046	-.0000634302
18	-.0025009878	.0003455492	-.0000478500
19	-.0018881264	.0002609535	-.0000361113
20	-.0014255162	.0001970521	-.0000272583
21	-.0010762808	.0001467916	-.0000205781
22	-.0008126172	.0001123476	-.0000155360
23	-.0006135507	.0000848287	-.0000117248
24	-.0004632519	.0000640498	-.0000088563
25	-.0003497721	.0000483605	-.0000066867

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 25  
 NUMBER OF RESPONSE FACTORS PER SET = 26  
 COMMON RATIO = .7550396065

TABLE 7 (Cont.)

## NEW RESFAC

RESPONSE FACTORS			
HOUR	Y	Y	Z
0	5.1759967596	.0000002177	.8567491015
1	-3.2693245749	.0001735577	-.5063326014
2	-.7627360929	.0019875912	-.1769113981
3	-.3954482093	.0053818007	-.0661057268
4	-.2236660857	.0077112659	-.0258009732
5	-.1372336213	.0082363143	-.0106121779
6	-.0902026578	.0075773935	-.0047087428
7	-.0624053974	.0064230733	-.0023146237
8	-.0446903991	.0051940988	-.0012785364
9	-.0327070448	.0040853314	-.0007867433
10	-.0242524529	.0031605263	-.0005255423
11	-.0181221788	.0024210053	-.0003703670
12	-.0134018519	.0018436045	-.0002693425
13	-.0102351466	.0013989911	-.0001993223
14	-.0077130395	.0010593995	-.0001488973
15	-.0058172469	.0008012595	-.0001117857
16	-.0043895015	.0006055851	-.0000841464
17	-.0033130656	.0004575046	-.0000634302
18	-.0025009877	.0003455492	-.0000479500
19	-.0018981263	.0002609535	-.0000361113
20	-.0014255152	.0001970521	-.0000272583
21	-.0010762807	.0001487916	-.0000205781
22	-.0008176172	.0001123476	-.0000155360
23	-.0006135507	.0000848287	-.0000117298
24	-.000442518	.0000640498	-.0000088563
25	-.0003497721	.0000483605	-.0000066667
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 25			
NUMBER OF RESPONSE FACTORS PER SET = 26			
COMMON RATIO = .7550396037			

TABLE 7 (Cont.)

## ROOTS

ROOT #	OLD	NEW
1	0.28098507216704	0.28098507582694
2	0.84228454393237	0.84228454471546
3	1.02137107973186	1.02137107926540
4	2.17346651520421	2.17346650715298
5	5.11260893999551	5.11260894227845
6	6.05598433406738	6.05598433479236
7	8.22009552003982	8.22009552583131
8	15.68213748921056	15.68213749230597
9	16.65349741351815	16.65349740592023
10	20.10396604661435	20.10396604274922
11	30.63091142382473	30.63091143841234

## B(ROOTS)

FOR ROOTS #	OLD	NEW
1	6.780432E-8	2.87200E-11
2	6.798700E-10	-2.00281E-9
3	-7.311400E-10	3.30000E-13
4	5.621767E-8	-7.90000E-13
5	8.845090E-9	-4.60900E-11
6	-3.960310E-9	-1.52034E-9
7	5.605072E-8	4.30000E-13
8	-9.308060E-9	-1.15000E-12
9	-1.537846E-8	3.25792E-9
10	3.210414E-8	-1.76000E-12
11	5.580300E-10	-9.69469E-9

TABLE 8

DESCRIPTION OF CONSTRUCTION							
LAYER NUMBER	THICKNESS FT	CONDUCTIVITY BTU PER (HR)(FT)(F)	DENSITY LB PER CU FT	SPECIFIC HEAT BTU PER (LB)(F)	RESISTANCE		
					(HR)(SQ FT)(F) PER BTU		
1	.0100	2.300	70.0	.3500	0.0000		ASPHALT SHINGLE
2	.0420	.067	34.0	.2900	0.0000		PLYWOOD SHEATHING
3	.2500	.025	.5	.1600	0.0000		INSULATION
4	0.0000	0.000	0.0	0.0000	.9100		AIR SPACE
5	.0420	.067	34.0	.2900	0.0000		PLYWOOD
6	.5000	.025	.5	.1600	0.0000		INSULATION
7	.0625	.420	100.0	.2000	0.0000		GYPSUM BOARD
8	0.0000	0.000	0.0	0.0000	.6850		INSIDE AIR

## OLD RESFAC

RESPONSE FACTORS			
HOUR	Y	Y	Z
0	.7251569790	.0003756736	.8374457518
1	-.6562617879	.0036660986	-.5326726953
2	-.0101950075	.0055020236	-.1754137625
3	-.0074798669	.0050052262	-.0546083780
4	-.0055072144	.0039969156	-.0211851227
5	-.0040565343	.0030453759	-.0082261789
6	-.0029385457	.0022765117	-.0036634230
7	-.0022019154	.0016879889	-.0019206027
8	-.0016223972	.0012472018	-.0011622678
9	-.0011954206	.0009200953	-.0007742682
10	-.0008908205	.0006783189	-.0005438346
11	-.0006490160	.0004999254	-.0003920535
12	-.0004782159	.0003683998	-.0002860650
13	-.0003523651	.0002714615	-.0002094686
14	-.0002596341	.0002000258	-.0001543414
15	-.0001913070	.0001473869	-.0001136275
16	-.0001409613	.0001086000	-.0000836932
17	-.0001038649	.0000800202	-.0000616578
18	-.0000765311	.0000509616	-.0000454282
19	-.0000563906	.0000434448	-.0000334719
20	-.0000415505	.0000320116	-.0000246629
21	-.0000306158	.0000235872	-.0000181723

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 21

NUMBER OF RESPONSE FACTORS PER SET = 22

COMMON RATIO = .7368329969

TABLE 8 (Cont.)

## NEW RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	.7251569715	.0003756761	.8374457390
1	-.65622617794	.0036660957	-.5326726411
2	-.0101950074	.0055020236	-.1759137619
3	-.0074798671	.0050052263	-.0596083779
4	-.0055072146	.0039969156	-.0211851230
5	-.0040565345	.0030453759	-.0082261792
6	-.0029885459	.0022765118	-.0036634233
7	-.0022019155	.0016879889	-.0019206029
8	-.0016223073	.0012472019	-.0011622680
9	-.0011954207	.0009200953	-.0007742684
10	-.0008308205	.0006783189	-.0005438347
11	-.0006490160	.0004999255	-.0003920536
12	-.0004782158	.0003683998	-.0002860651
13	-.0003523651	.0002714615	-.0002098687
14	-.0002596342	.0002000259	-.0001543415
15	-.0001913070	.0001473870	-.0001136275
16	-.0001409613	.0001086000	-.0000836932
17	-.0001038649	.0000800202	-.0000616578
18	-.0000765311	.0000589616	-.0000454282
19	-.0000563997	.0000434443	-.0000334719
20	-.0000415595	.0000320116	-.0000246629
21	-.0000306158	.0000235672	-.0000181723
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 21			
NUMBER OF RESPONSE FACTORS PER SET = 22			
COMMON RATIO = .7368330005			

TABLE 8 (Cont.)

## ROOTS

ROOT #	OLD	NEW
1	0.30539401103140	0.30539400613318
2	1.12468525501743	1.12468526089836
3	9.41824542784383	9.41824542612801
4	12.27380260558755	12.27380260257945
5	33.85678581614047	33.85678582246646

## B(ROOTS)

FOR ROOTS #	OLD	NEW
1	-3.4314850E-7	2.93000E-12
2	-2.9825105E-7	-1.36000E-12
3	-5.2242660E-8	-7.50152E-9
4	7.2314770E-8	-1.06420E-10
5	6.6737980E-8	3.69000E-12

TABLE 9

DESCRIPTION OF CONSTRUCTION						
LAYER NUMBER	THICKNESS FT	CONDUCTIVITY BTU PER (HR)(FT)(F)	DENSITY LB PER CU FT	SPECIFIC HEAT BTU PER (LB)(F)	RESISTANCE (HR)(SQ FT)(F) PER BTU	
1	.0417	.830	55.0	.4000	0.0000	STONE
2	.0313	.110	70.0	.4000	0.0000	FELT
3	.1670	.025	2.0	.2000	0.0000	CELLULAR GLASS
4	.3330	.100	40.0	.2000	0.0000	CONCRETE
5	.0050	26.000	480.0	.1000	0.0000	METAL PAN
6	0.0000	0.000	0.0	0.0000	.9100	AIR SPACE
7	.0625	.420	100.0	.2000	0.0000	GYPSUM BOARD
8	0.0000	0.000	0.0	0.0000	.9100	AIR SPACE
9	.0625	.035	30.0	.2000	0.0000	ACOUSTICAL TILE
10	0.0000	0.000	0.0	0.0000	.6850	INSIDE AIR

## OLD RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	.7578139001	.0027676836	.8455636399
1	-.3264110204	.0257121522	-.3394234703
2	-.0551369293	.0378060569	-.1224280305
3	-.0406467104	.0340939470	-.0561552755
4	-.0303621399	.0272297798	-.0321965209
5	-.0227925210	.0209383902	-.0214183746
6	-.0171418960	.0158870616	-.0153359574
7	-.0129010796	.0119958160	-.0113254352
8	-.0097119218	.0090413992	-.0084651492
9	-.0073118274	.0068100785	-.0063561928
10	-.0055050614	.0051281546	-.0047808134
11	-.0041448055	.0038612698	-.0035981829
12	-.0031206732	.0029072631	-.0027087421
13	-.0023495961	.0021889357	-.0020393435
14	-.0017690432	.0016480851	-.0015354205
15	-.0013319373	.0012408677	-.0011560313

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 15  
 NUMBER OF RESPONSE FACTORS PER SET = 16  
 COMMON RATIO = .7529140452



TABLE 9 (Cont.)

## NEW RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	.7574138940	.0027677014	.8455636324
1	-.3264110193	.0257171323	-.3394234438
2	-.0551369245	.0378860570	-.1724280295
3	-.0406467189	.0340939471	-.0561332749
4	-.0303621396	.0272297800	-.0371965207
5	-.0227925708	.0209383903	-.0214187745
6	-.0171418959	.0158870618	-.0153359574
7	-.0129010796	.0119958162	-.0113254352
8	-.0097119219	.0090413994	-.0084651493
9	-.0073118275	.0068100787	-.0063561929
10	-.0055050615	.0051281548	-.0047808135
11	-.0041448056	.0038612700	-.0035991830
12	-.0031206713	.0029072632	-.0027087472
13	-.0023449962	.0021889358	-.0020393435
14	-.0017690433	.0016480851	-.0015354206
15	-.0013319373	.0012408677	-.0011560314
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 15			
NUMBER OF RESPONSE FACTORS PER SET = 14			
COMMON RATIO = .75291404E3			

## ROOTS

ROOT #	OLD	NEW
1	0.28380420751416	0.28380420343971
2	1.27316441620625	1.27316442200970
3	6.08916372121456	6.08916372270650
4	16.59531217201106	16.59531217589722
5	31.14535162877291	31.14535163507094

## B (ROOTS)

FOR ROOTS #	OLD	NEW
1	=4.681876E-8	1.800E-12
2	=4.410708E-8	-3.000E-14
3	1.978496E-8	-1.238E-11
4	=5.435033E-8	-8.071E-11
5	3.951397E-8	1.320E-12

TABLE 10

DESCRIPTION OF CONSTRUCTION							
LAYER NUMBER	THICKNESS FT	CONDUCTIVITY BTU PER (HR) (FT) (F)	DENSITY LB PER CU FT	SPECIFIC HEAT BTU PER (LB) (F)	RESISTANCE (HR) (SQ FT) (F)	PER BTU	
1	.0050	26.000	480.0	.1000	0.0000		METAL SIDING
2	.0420	.025	2.0	.2000	0.0000		INSULATION PANELS
3	.0833	.400	116.0	.2000	0.0000		STUCCO
4	.0313	.110	70.0	.4000	0.0000		FELT
5	.0420	.067	34.0	.2900	0.0000		PLYWOOD SHEATHING
6	0.0000	0.000	0.0	0.0000	.9100		AIR SPACE
7	.0625	.420	100.0	.2000	0.0000		GYPSON BOARD
8	.0104	.070	37.0	.6000	0.0000		PANELING
9	0.0000	0.000	0.0	0.0000	.6850		INSIDE AIR

## OLD RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	1.8963060277	.0000004210	.4011426131
1	-1.7737519020	.0001551499	-.1512127477
2	-.0093253891	.0012842006	-.0296006032
3	-.0055364512	.0028719285	-.0201942048
4	-.0043045688	.0038685829	-.0152586240
5	-.0035601171	.0042707124	-.0122500417
6	-.0030526487	.0043131968	-.0102674603
7	-.0026786892	.0041663052	-.0088693871
8	-.0023856690	.0039289601	-.0078185899
9	-.0021447051	.0036553284	-.0069833746
10	-.0019393478	.0033741921	-.0062892735
11	-.0017599490	.0031003295	-.0056933503
12	-.0016006431	.0028408834	-.0051701766
13	-.0014576938	.0025988771	-.0047041173
14	-.0013285809	.0023751441	-.0042950716
15	-.0012114942	.0021693853	-.0039061168
16	-.0011050518	.0019807441	-.0035621987
17	-.0010081407	.0018081172	-.0032494010
18	-.0009198274	.0016503210	-.0029645329
19	-.0008393047	.0015061780	-.0027048935
20	-.0007659611	.0013745599	-.0024681341
21	-.0006988606	.0012544076	-.0022521756
22	-.0006377307	.0011447383	-.0020551558
23	-.0005819529	.0010446462	-.0018753946
24	-.0005310563	.0009532999	-.0017113698
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 24			
NUMBER OF RESPONSE FACTORS PER SET = 25			
COMMON RATIO = .9125489730			

TABLE 10 (Cont.)

## NEW RESFAC

RESPONSE FACTORS			
HOUR	X	Y	Z
0	1.8963060503	.0000003904	.4011426325
1	-1.7737519229	.0001551815	-.1512127661
2	-.0093253903	.0012842006	-.0299006033
3	-.0055369520	.0028719285	-.0201942051
4	-.0043045492	.0038685829	-.0152586282
5	-.0035601173	.0042707124	-.0122500420
6	-.0030526499	.0043131969	-.0102674505
7	-.0024786993	.0041663052	-.0088693873
8	-.0023856490	.0039289601	-.0078185901
9	-.0021447051	.0036553284	-.0069833747
10	-.0019393478	.0033741921	-.0062892736
11	-.0017599499	.0031003295	-.0056933503
12	-.0016006431	.0028408834	-.0051701767
13	-.0014576938	.0025988770	-.0047041173
14	-.0013285808	.0023751441	-.0042750717
15	-.0012114942	.0021693853	-.0039051168
16	-.0011050517	.0019807440	-.0035621987
17	-.0010091407	.0018081172	-.0032749401
18	-.0009198773	.0016503210	-.0029645329
19	-.0008199047	.0015061780	-.0027048935
20	-.0007658610	.0013745599	-.0024681341
21	-.0006989606	.0012544075	-.0022521756
22	-.0006377307	.0011447382	-.0020551558
23	-.0005919529	.0010446462	-.0018753946
24	-.0005310543	.0009532999	-.0017113698
NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 24			
NUMBER OF RESPONSE FACTORS PER SET = 25			
COMMON RATIO = .9125469711			

TABLE 10 (Cont.)

ROOTS		
ROOT #	OLD	NEW
1	0.09151352592139	0.09151352798700
2	0.59670697196906	0.59670696614259
3	1.87826524925806	1.87826525339801
4	4.46784784597435	4.46784784772299
5	5.21288150152188	5.21288150553909
6	6.84838752278688	6.84838751809579
7	9.19016020960470	9.19016020792799
8	15.55378495246231	15.55378495460405
9	21.54078931780691	21.54078932206482
10	25.48150421908758	25.48150421445916
11	26.15232440552245	26.15232441059038
12	36.39218806754798	36.39218806540566

B (ROOTS)		
FOR ROOTS #	OLD	NEW
1	2.4113701E-7	1.50000E-13
2	2.6229038E-7	-2.40000E-13
3	1.0058861E-7	-9.31998E-9
4	-1.3448400E-9	4.28628E-9
5	-8.6910700E-9	1.92000E-12
6	1.8604440E-8	-2.90600E-11
7	-1.0867190E-8	7.27528E-9
8	-4.9799730E-8	1.23000E-12
9	2.9246420E-8	-2.29313E-9
10	5.6764700E-9	9.40000E-13
11	7.1909600E-9	2.68070E-10
12	3.9220780E-8	-5.25500E-11

## APPENDIX C

### BISECTION METHOD FOR LOCATING ROOTS

Given a continuous function  $f$  on the interval  $[a, b]$ , where  $f(a)$  and  $f(b)$  have opposite signs. To find a solution to  $f(x) = 0.0$ :

- 1) set  $a_1 = a$  and  $b_1 = b$
- 2) set  $i = 1$
- 3) set  $p_i = \frac{1}{2}(a_i + b_i)$
- 4) if  $\text{ABS}(f(p_i)) \leq 1.0\text{E-}8$  go to 6, else go to 5
- 5) if  $f(p_i) * f(a_i) > 0$ , set  $a_{i+1} = p_i$  and  $b_{i+1} = b_i$ ;  
if  $f(p_i) * f(a_i) < 0$ , set  $a_{i+1} = a_i$  and  $b_{i+1} = p_i$ ;  
add 1 to  $i$  and go to 3
- 6) end

## APPENDIX D

### SECANT METHOD FOR LOCATING ROOTS

Given a continuous function  $f$  on the interval  $[p_0, p_1]$ , where  $p_0$  and  $p_1$  are initial root approximations such that  $p_0 \neq p_1$ . To find a solution to  $f(x) = 0.0$ :

1) set  $i = 2$

2) set  $p_i = p_{i-1} - \frac{f(p_{i-1}) * (p_{i-1} - p_{i-2})}{f(p_{i-1}) - f(p_{i-2})}$

3) if  $ABS(f(p_i)) \leq 1.0E-8$  go to 4, else add 1 to  $i$  and go to 2

4) end

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16. Abstract <p>The RESFAC2 version of the Thermal Response Factor Program (RESFAC) is the result of numerous modifications and additions to the original RESFAC. These modifications and additions have significantly reduced the program's computational time requirement. As a result of this work, the program is more efficient and its code is both readable and understandable.</p> <p>This report describes what a thermal response factor is; analyzes the original matrix algebra calculations and root finding technique; presents a new root finding technique and streamlined matrix algebra; supplies ten validation cases and their results.</p>					
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